Integration Review

Definition of the definite integral:

Let let $f(x)$ be continuous on $[a, b]$ and $x_0, x_1, \ldots, x_n$ be the endpoints of n equal subdivisions of the interval $[a, b]$, with $x_0 = a$ and $x_n = b$. Set Δx be the length of each subdivision, so $\Delta x = \frac{b-a}{n}$. For each $1 \leq i \leq n$, let x_i^* be any point in the subinterval $[x_{i-1}, x_i]$. Then *the definite integral of $f(x)$ from a to b* is

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x.$$

This value measures the amount of net (signed) area from a to b between f and the x-axis.

FTC part 2: $\int_a^b f(x)\,dx = F(b) - F(a)$ where $F(x)$ is an antiderivative of $f(x)$.

FTC part 1: $F(x) = \int_a^x f(t)\,dt \implies F'(x) = f(x)$.

Basic indefinite integrals (antiderivatives), C is any constant:

$\int x^p\,dx = \frac{1}{p+1}x^{p+1} + C, \quad p \neq -1$

$\int e^x\,dx = e^x + C$

$\int \frac{1}{x}\,dx = \ln|x| + C$

$\int a^x = \frac{1}{\ln a}a^x + C, \quad a > 0$

$\int \cos(x)\,dx = \sin(x) + C$

$\int \sin(x)\,dx = -\cos(x) + C$

$\int \sec^2(x)\,dx = \tan(x) + C$

$\int \csc^2(x)\,dx = -\cot(x) + C$

$\int \tan(x)\sec(x)\,dx = \sec(x) + C$

$\int \cot(x)\csc(x)\,dx = -\csc(x) + C$

$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin(x) + C$

$\int \frac{1}{1+x^2}\,dx = \arctan(x) + C$

Properties of integrals:

Linearity: $\int (f(x) + g(x))\,dx = \int f(x)\,dx + \int g(x)\,dx$

$\int cf(x)\,dx = c\int f(x)\,dx, \quad c$ any constant

Switching limits: $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

Breaking up integrals: If c is any point in $[a, b]$, then $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

Comparison properties: If $f(x) \geq 0$ for $a \leq x \leq b$, then $\int_a^b f(x)\,dx \geq 0$

If $f(x) \leq g(x)$ for $a \leq x \leq b$, then $\int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx$